LIQUID PRODUCT SPRAYING IN A FLUIDIZED BED

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A method is given for calculating the characteristics of a bed of product falling upon the particles in the fluidized bed from the spraying of the liquid product into the bed.

Some processes in chemical and food technology require the injection of a liquid product into a fluidized bed. These include, for example, the drying of liquid products on inert bodies and granulation in fluidized beds [1, 2]. We consider the case where the input is by spraying into the bed. The input is local in the sense that the product can strike the particles only in the injection zone around the air jet produced by the pneumatic injector. In order to calculate such processes, one has to analyze the distribution of the characteristics in the layer of product deposited on a particle in a single passage through the injection zone. The deposition of droplets on a particle is of random character, and a random number of droplets of polydisperse composition may fall at a certain point on the particle. Therefore, the thickness of the liquid layer is determined not only by the dispersion in the spray and by the flow of the droplets over the particle but also by the possibility that droplets fuse.

When the droplet stream encounters the particles within the jet, the droplets are captured by the particles, and the density of the droplet flow decreases as the jet penetrates into the layer. This absorption of the droplets in a fluidized bed has been considered in [3, 4] on the basis of absorbing-particle concepts, and in [5] it was discussed by means of a balance equation for the number of particles in an elementary layer of the bed. The latter equation for one-dimensional motion can be put in the form

$$\frac{dn_{\mathrm{d}}}{dx} = -\sigma_{\mathrm{p}} n_{\mathrm{p}} n_{\mathrm{d}} \tag{1}$$

on the basis that the speeds of the particles in the injection zone are much less than the droplet speeds. We take n_{p} as constant, and then we can show that the number of droplets striking a particle with coordinate x in unit time is

$$\lambda_{\mathrm{d}}(x) = \frac{q_m \sigma_{\mathrm{p}}}{\bar{m}_{\mathrm{d}}} \exp(-n_{\mathrm{p}} \sigma_{\mathrm{p}} x). \tag{2}$$

If we assume that the individual droplets strike the particle instantaneously, and also that the individual droplets are formed independently, we have a point flux of events consisting of droplets falling on particles, which is a Poisson flux [6] and has intensity λ_{d}.

We now consider the change in thickness z of the layer of product at an arbitrary point on a particle with coordinate x. Here z is the current thickness of the product at time τ. Interest attaches to the thickness of the newly deposited layer, so $z = 0$ at $\tau = 0$, which corresponds to the particle entering the injection zone. The layer thickness may alter stepwise by random quantity Δz at random instants corresponding to droplets striking the particle. As the sizes of the droplets are independent of one another and the flow is of Poisson type, the random thickness change over time is a discontinuous mark process and is described by the Kolmogorov–Feller equation [6]

$$\frac{\partial f}{\partial \tau} = -\lambda_{\mathrm{d}} f + \lambda_{\mathrm{d}} \int_0^{\infty} b(z - z') f(z')\, dz'. \tag{3}$$

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We note that when a drop strikes a particle, it does not necessarily coat the point on the surface, so there is a finite probability that $\Delta z = 0$. Therefore, the $b(\Delta z)$ distribution has a δ singularity at zero [6]. In deriving (3) it has been assumed that the thickness is not reduced by the drying, since the passage of a particle through the injection zone lasts only a fraction of a second.

We assume that the droplets are small and that there is a relationship of the form $f_d = c\Delta z_d^2$ between the area that they cover and the thickness Δz_d of a spread droplet; this follows from the fact that the spreading droplets are geometrically similar one to another. Therefore, the thickness of a spread droplet is uniquely related to the diameter, and Δz_d can be taken as the characteristic size of the droplets.

In general, not all points on the surface are equally accessible. For a spherical particle in which the radius of the sphere forms an angle θ with the direction of the relative motion of the droplets, the probability of coating by some droplet striking the particle is

$$p_c = \frac{f_d}{\sigma_p} c(\theta), \tag{4}$$

where

$$c(\theta) = \begin{cases} \cos\theta & \text{for} \quad 0 \leqslant \theta \leqslant \frac{\pi}{2}, \\ 0 & \text{for} \quad \frac{\pi}{2} \leqslant \theta \leqslant \pi. \end{cases} \tag{5}$$

When a droplet strikes a particle, there may be two outcomes: the droplet covers the point and the layer thickness is increased by $\Delta z = \Delta z_d$, or else the droplet does not cover the point, and the layer is unaltered, i.e., $\Delta z = 0$. If the thickness of the spread drop is Δz_d, then the probability density is

$$b_y(\Delta z|\Delta z_d) = p_c \delta(\Delta z - \Delta z_d) + (1 - p_c)\delta(\Delta z). \tag{6}$$

Then the following is the probability density for the layer thickness increasing by Δz when one drop strikes a particle on the basis of the density distribution for the spread droplet thicknesses:

$$b(\Delta z) = \int_0^\infty b_y(\Delta z|\Delta z_d) g_d(\Delta z_d) d(\Delta z_d). \tag{7}$$

We substitute (2) and (7) into (3) and perform a Fourier transformation to get

$$\frac{\partial\chi}{\partial\tau} = \frac{q_m \exp(-n_p\sigma_p x) c(\theta) i[\varphi''(s) - \varphi''(0)]}{\rho\varphi'''(0)} \chi \tag{8}$$

Then (8) describes the random growth of the product layer at a point on the surface with angular coordinate θ for a particle with coordinate x. To solve this equation we require data on the motion of the bed particles in the injection zone. We consider some limiting cases.

We assume that the particles in the injection zone move uniformly parallel to the direction of droplet motion or perpendicular to it. In the first case, the coordinates of a particle in the injection zone vary linearly, while in the second they are random for a set of particles but constant for a single occurrence of a particle in the injection zone. Then let the x coordinate have a uniform distribution with probability density $f_x = x_{in}^{-1}$. We assume as regards particle rotation that either the rotation is rapid or is absent. In the latter case we have in mind that the angle θ is a random quantity for the set of particles. As all points on the surface are equivalent, it can be shown that θ has the distribution $f_\theta = 1/2 \sin\theta$.

We combine these cases to get four forms of motion in the injection zone. We average the solution to (8) on the basis of the distributions for x and θ to derive expressions for the characteristic thickness of the product layer at a point on the surface deposited in a single transit through the injection zone. We use the relationship between the derivatives of the characteristic function and the statistical moments of the random quantity [6] to derive relationships for the mathematical expectation of the thickness z and the second moment about the origin (mean square):

$$\overline{z} = \frac{q_m \tau_{irr}}{4\sigma_p n_p \varkappa_{irr}}, \quad (9)$$

$$\alpha_2(z) = \frac{\overline{z}\alpha_{4d}(1+nA_1)}{\alpha_{3d}}. \quad (10)$$

The values of the coefficient n are dependent on the particle motion in the injection zone and are given in Table 1. We considered the above forms of particle motion for constant time spent in the injection zone and also the first case for an exponential distribution of the time spent in that zone.

We note that there is a finite probability that no drop will strike the working point on the surface. Therefore, the probability density f(z) takes the form

$$f(z) = (1 - a_1)\delta(z) + a_1 f^*(z). \quad (11)$$

We perform a Fourier transformation on (11) and use the fact that f*(z) is the probability density for a continuous random quantity to get that

$$a_1 = 1 - \lim_{s\to\infty} \chi(s). \quad (12)$$

This formula gives us expressions for the A_2 dependence of the probability a_1, which is a geometrical probability and is thus simultaneously equal to the mean fraction of the particle surface covered by the product in the injection zone (Table 1).

As there is a finite probability that no drop reaches a point on the surface, the averaging in (9) and (10) includes the case z = 0. The characteristics of the layer of product can be obtained subject to the conditions $z \neq 0$ by using the probability density f*(z). Then we get the following expressions for the mathematical expectation for the layer thickness and the coefficient of variation of this:

$$\overline{z^*} = \frac{\alpha_{3d} A_2}{\alpha_{2d} a_1(A_2)}, \quad (13)$$

$$v_{z^*} = \left(\frac{\alpha_{4d}\alpha_{2d} a_1 (1+nA_1)}{\alpha_{3d}^2 A_2} - 1\right)^{\frac{1}{2}}. \quad (14)$$

It can be shown that (9) implies a formula for the mathematical expectation of the layer thickness:

$$\overline{z} = G/\rho \dot{F}_p. \quad (15)$$

These formulas show that the probability of several droplets reaching a single point is dependent on the ratio of the average liquid layer thickness to some characteristic thickness of the layer formed by the spread of a single drop. This ratio is expressed from the dimensionless quantities A_1 and A_2, which may be called discrete deposition factors. In accordance with (15), the mean thickness is equal to the ratio of the output from the sprayer to the surface of the particles passing through the injection zone in unit time. The characteristic drop thickness of appearing in A_1 and A_2 are the mean-mass value α_{4d}/α_{3d} and the mean volume-surface one α_{3d}/α_{2d}.

We performed calculations on the dependence of the probability a_1 on the factor A_2 (Fig. 1), and also on the ratio of the mathematical expectation of the layer thickness to the characteristic droplet thickness (Fig. 2) as a function of the same, together with the coefficient of variation for the layer thickness for monodisperse droplets (Fig. 3). The integrals in the formulas given in Table 1 were calculated numerically.

The relationships show that when A_2 is small there is usually not more than one droplet reaching a point on the surface in a single passage of a particle through the injection zone. At the same time, the average proportion of the particle surface coated by the product is small. Also, a_1 increases with A_2, and there are also increases in the mean thickness and the coefficient of variation for the thickness, which is due to the increase in the probability that several droplets will reach a given point. Therefore, the thickness distribution is substantially polydisperse even if the initial droplets are monodisperse. An important point is that relatively complete coating can be attained along with an increase in layer thickness

TABLE 1. Forms of Particle Motion in the Injection Zone

For number	Particle displacement relative to droplet motion direction	Particle rotation	Residence time	n	a_1
1	Parallel	Rotates	Constant	1	$1-\exp(-A_2)$
2	Perpendicular	Rotates	Constant	$\frac{B}{2}$	$1-\int_0^1 \exp[-A_2 B \exp(-B\xi)]\, d\xi$
3	Parallel	Does not rotate	Constant	$\frac{B}{3}$	$\frac{4A_2-1+\exp(-4A_2)}{8A_2}$
4	Perpendicular	Does not rotate	Constant	$\frac{4B}{3}$	$\frac{1}{2}\left[1-\int_0^1 \frac{1-\exp(-4A_2 B\exp(-B\xi))}{4A_2 B\exp(-B\xi)} d\xi\right]$
5	Parallel	Rotates	Random	2	$\frac{A_2}{1+A_2}$

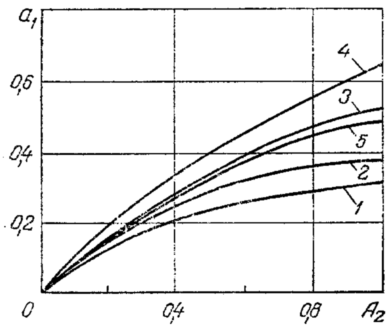

Fig. 1

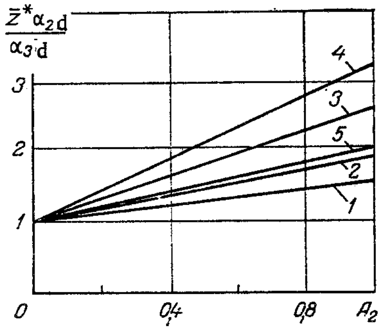

Fig. 2

Fig. 1. Dependence on the deposition discreteness factor A_2 for the probability a_1 of at least one droplet reaching a point on the surface of a particle for various forms of particle motion (1-5 see Table 1).

Fig. 2. Dependence of $\bar{z}^*\alpha_{2d}/\alpha_{3d}$, the ratio of the mathematical expectation for the deposited layer thickness to the characteristic drop thickness, on the discreteness factor A_2 for various forms of particle motion (1-5 see Table 1).

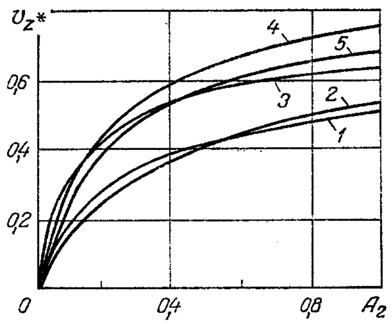

Fig. 3. Dependence of the coefficient of variation for the thickness v_{z^*} on the discreteness factor A_2 for various forms of particle motion (1-5 see Table 1) for monodispersed droplets.

at the same time as the polydispersion increases. As the drying in this polydisperse layer is uneven, droplets may strike the still wet surface on subsequent entry of the particle into the injection zone. On the other hand, when a_1 is small the time between two successive depositions on a given part of the surface will show considerable fluctuations even if there is fairly regular particle entry into the injection zone [7]. It is therefore important to consider the repeated passage through the injection zone in the organization of liquid input to fluidized beds.

NOTATION

n_d, n_p, numbers of drops and particles per unit volume; x, distance to the initial irrigation surface; σ_p, mean area of projection on a plane; λ_d, number of drops falling on a particle per unit time; q_m, mass flow density of drops on the initial irrigation surface; $\bar{m}_d$, mean mass of a drop; z, bed thickness; Δz, increment in bed thickness; Δz_d, thickness of a spreading drop; f(z), b(Δz), g-(Δz_d), probability densities of z, Δz, Δz_d; τ, time; f_d, area covered by spreading drop; c, coefficient; p_c, probability of covering a given point on the surface by a drop falling on a particle; θ, angular coordinate; $\delta(\ )$, delta function; $\chi(s)$, characteristic functions of z and Δz_d; ρ, product density; x_{in}, input zone depth; τ_{ir}, irrigation time; $\bar{z}$, $\alpha_2(z)$, mathematical expectation and second origin moment of z; α_{3d}, α_{4d}, second, third, and fourth origin moments of Δz_d; A_1, A_2, deposition discreteness factors; α_1, probability of covering a point by a single drop; $f^*(z)$, probability density of z for $z \neq 0$; B, dimensionless injection zone depth; $\bar{z}^*$, v_{z^*}, mathematical expectation and coefficient of variation for z for $z \neq 0$, G, sprayer efficiency; F_p, area of particles passing through the zone per unit time.

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